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Investigation of the scattering of anti-plane shear waves by two collinear cracks in a piezoelectric material using a new method

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Summary. In this paper, the dynamic interaction between two collinear cracks in a piezoelectric material under harmonic anti-plane shear waves is investigated. By using the Fourier transform, the problem can be solved with two pairs of triple integral equations. These equations are solved using Schmidt's method. This process is quite different from that adopted previously. Numerical examples are provided to show the effect of the geometry of the interacting cracks, the shear stress wave velocity of the piezoelectric materials, and the frequency of the incident wave upon the dynamic stress intensity factor of the cracks.

1 Introduction

It is well known that piezoelectric materials produce an electric field when deformed and undergo deformation when subjected to an electric field. The coupling nature of piezoelectric materials has attracted wide applications in electric-mechanical and electric devices, such as electric-mechanical actuators, sensors and structures. When subjected to mechanical and electrical loads in service, these piezoelectric materials can fail prematurely due to defects, e.g. cracks, holes, etc. arising during their manufacture process. Therefore, it is of great importance to study the electro-elastic interaction and fracture behaviors of piezoelectric materials. Moreover, it is known that the failure of solids results from the final propagation of the crack, and in most cases, the unstable growth of the crack is brought about by the external dynamic loads. So, the study of the dynamic fracture mechanics of piezoelectric materials is much more urgent in recent research.

Recently, the dynamic response of piezoelectric materials and the failure modes have attracted more and more attention from many researchers ([9], [12], [17], [18], [23]). A finite crack in an infinite piezoelectric material strip under anti-plane dynamic electromechanical impact was investigated with the well-established integral transform methodology [23]. Axisymmetric vibration of a piezo-composite hollow cylinder was studied by Paul and Nelson [17]. The dynamic representation formulae and fundamental solutions for piezoelectricity had been proposed earlier by Khutoryansky and Sosa [9]. The dynamic response of a cracked dielectric medium in a uniform electric field was studied by Shindo et al. [18]. Narita and Shindo [12] also carried out an analysis of the scattering of anti-plane shear waves by a finite crack in piezoelectric laminates. However, the dynamic behavior of two cracks in piezoelectric materials has not been studied.

In the present paper, the interaction between two collinear symmetrical cracks subject to harmonic anti-plane shear waves in piezoelectric materials was investigated using a somewhat different approach by a new method, namely Schmidt's method [10]. It is a simple and convenient method for solving this problem. Fourier transform is applied, and a mixed boundary value problem is reduced to two pairs of triple integral equations. In solving the triple integral equations, the crack surface displacement and the electric potential are expanded in a series using Jacobi's polynomials, and Schmidt's method [10] is used. This process is quite different from that adopted in references [2], [4], [6], [9], [12]–[18], [20]–[25]. The form of solution is easy to understand. The main purpose of the article is to dvelop a new theoretical model to investigate the dynamic behavior of piezoelectric materials. Accordingly, the paper addresses the different algorithms. The theoretical formulations governing the steady-state problem are based upon the use of integral transform techniques. The resulting dynamic stress intensity factors at the interacting cracks are obtained by Schmidt's method. Numerical examples are provided to show the effect of the geometry of the interacting cracks, the shear stress wave velocity of the piezoelectric materials and the frequency of the incident wave upon the resulting dynamic stress intensity factors.

2 Formulation of the problem

Consider an infinite piezoelectric body containing two collinear symmetric cracks of length 1-b along the x-axis. $2b$ is the distance between two cracks. The piezoelectric boundary-value problem for anti-plane shear is considerably simplified if we consider only the out-of-plane displacement and the in-plane elastic fields, see Fig. 1. Let ω be the circular frequency of the incident wave. In what follows, the time dependence of all field quantities assumed to be of the form $\exp(-i\omega t)$ will be suppressed but understand. We further suppose that the two faces of the crack do not come into contact during vibrations. The constitutive equations can be written as

$$\tau_{zk} = c_{44} w_{,k} + e_{15} \phi_{,k} , \tag{1}$$

$$D_k = e_{15} w_{,k} - \varepsilon_{11} \phi_{,k} , \tag{2}$$

where $\tau_{zk}, D_k (k = x, y)$ are the anti-plane shear stress and in-plane electric displacement, respectively. $c_{44}, e_{15}, \varepsilon_{11}$ are the shear modulus, piezoelectric coefficient and dielectric parameter, respectively. w and ϕ are the mechanical displacement and electric potential.

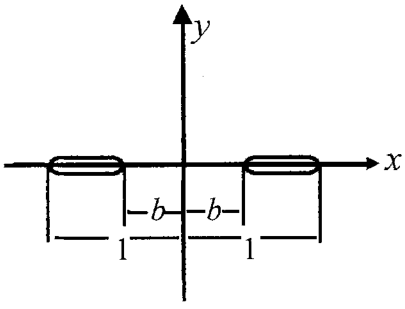

Fig. 1. Collinear cracks in a piezoelectric material body

The anti-plane governing equations for piezoelectric materials are [12]

$$c_{44}\nabla^2 w + e_{15}\nabla^2\phi = \varrho\partial^2 w/\partial t^2\,, \tag{3}$$

$$e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2\phi = 0\,, \tag{4}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator and ϱ is the mass density of the piezoelectric materials. Body force, other than inertia, and the free charge are ignored in the present work. It is worth mentioning that the field equations (3) and (4) for anti-plane deformation can also be derived by considering the so-called Bleustein-Gulayev SH waves. Because of the assumed symmetry in geometry and loading, it is sufficient to consider the problem for $0 \le x \le \infty, 0 \le y \le \infty$ only.

A Fourier transform is applied to Eqs. (3) and (4). Assume that the solution is

$$\bar{w}(s, y, t) = A(s)e^{-\gamma y}\,, \tag{5}$$

where $\gamma = \sqrt{s^2 - (\omega/c_{SH})^2}$, $c_{SH} = \sqrt{\mu/\varrho}$, $\mu = c_{44} + \dfrac{e_{15}^2}{\varepsilon_{11}}$. c_{SH} is the shear stress wave velocity of the piezoelectric materials. $A(s)$ is an unknown function, and a superposed bar indicates the Fourier transform throughout the paper, e.g.,

$$\bar{f}(s) = \int_{-\infty}^{\infty} f(x)e^{-isx}\,dx\,. \tag{6}$$

Inserting Eq. (5) into Eq. (4), it can be assumed

$$\bar{\phi}(s, y, t) - \frac{e_{15}}{\varepsilon_{11}}\,\bar{w}(s, y, t) = B(s)\,e^{-sy}\,, \tag{7}$$

where $B(s)$ is an unknown function.

As discussed in Narita's [12], Shindo's [19] and Yu's [23] references, the boundary conditions of the present problem are:

$$\tau_{yz}(x, 0, t) = -\tau_0\,, \qquad b \le |x| \le 1\,, \tag{8}$$

$$D_y(x, 0, t) = -D_0\,, \qquad b \le |x| \le 1\,, \tag{9}$$

$$w(x, 0, t) = \phi(x, 0, t) = 0\,, \qquad |x| < b\,, \quad |x| > 1\,, \tag{10}$$

$$w(x, y, t) = \phi(x, y, t) = 0\,, \quad \text{for} \quad (x^2 + y^2)^{1/2} \to \infty\,. \tag{11}$$

In this paper, the wave is vertically incident, and we only consider that τ_0 and D_0 are positive. The problem therefore reduces to the determination of the unknown functions $A(s)$ and $B(s)$. Because of symmetry, the boundary conditions can be applied to yield two pairs of triple integral equations:

$$\frac{2}{\pi}\int_0^\infty A(s)\cos(sx)\,ds = 0 \qquad 0 \le x < b \quad \text{and} \quad x > 1\,, \tag{12}$$

$$\frac{2}{\pi}\int_0^\infty \gamma A(s)\cos(sx)\,ds = \frac{1}{\mu}\left(\tau_0 + \frac{e_{15}D_0}{\varepsilon_{11}}\right) \qquad b \le x \le 1\,, \tag{13}$$

and

$$\frac{2}{\pi}\int_0^\infty B(s)\cos(sx)\,ds = 0 \qquad 0 \le x < b \quad \text{and} \quad x > 1\,, \tag{14}$$

$$\frac{2}{\pi}\int_0^\infty sB(s)\cos(sx)\,ds = -\frac{D_0}{\varepsilon_{11}} \qquad b \le x \le 1\,. \tag{15}$$

To determine the unknown functions $A(s), B(s)$, the above two pairs of triple integral equations (12)–(15) must be solved.

3 Solution of the triple integral equation

For solving the above two pairs of triple integral equations, Schmidt's method [10] can be used to solve the triple integral equations (12)–(15). The displacement w and the electric potential ϕ can be represented by the following series:

$$w(x,0,t) = \sum_{n=0}^{\infty} a_n P_n^{(\frac{1}{2},\frac{1}{2})}\left(\frac{x-\frac{1+b}{2}}{\frac{1-b}{2}}\right)\left(1-\frac{\left(x-\frac{1+b}{2}\right)^2}{\left(\frac{1-b}{2}\right)^2}\right)^{\frac{1}{2}}, \quad \text{for} \quad b \le x \le 1\,,\ y=0\,, \tag{16}$$

$$w(x,0,t) = 0\,, \quad \text{for} \quad x > 1\,, \quad x < b\,, \quad y = 0\,, \tag{17}$$

$$\phi(x,0,t) = \sum_{n=0}^{\infty} b_n P_n^{(\frac{1}{2},\frac{1}{2})}\left(\frac{x-\frac{1+b}{2}}{\frac{1-b}{2}}\right)\left(1-\frac{\left(x-\frac{1+b}{2}\right)^2}{\left(\frac{1-b}{2}\right)^2}\right)^{\frac{1}{2}}, \quad \text{for} \quad b \le x \le 1\,,\ y=0\,, \tag{18}$$

$$\phi(x,0,t) = 0\,, \quad \text{for} \quad x > 1\,, \quad x < b\,, \quad y = 0\,, \tag{19}$$

where a_n and b_n are unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial [5]. The Fourier transformation of Eqs. (16) and (18) is [3]

$$A(s) = \bar{w}(s,0,t) = \sum_{n=0}^{\infty} a_n B_n G_n(s) \frac{1}{s} J_{n+1}\left(s\,\frac{1-b}{2}\right), \tag{20}$$

$$B(s) = \bar{\phi}(s,0,t) - \frac{e_{15}}{\varepsilon_{11}}\,\bar{w}(s,0,t) = \sum_{n=0}^{\infty}\left(b_n - \frac{e_{15}}{\varepsilon_{11}}\,a_n\right) B_n G_n(s) \frac{1}{s} J_{n+1}\left(s\,\frac{1-b}{2}\right), \tag{21}$$

$$B_n = 2\sqrt{\pi}\,\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{n!}\,, \tag{22}$$

$$G_n(s) = \begin{cases} (-1)^{\frac{n}{2}}\cos\left(s\,\frac{1+b}{2}\right), & n = 0,2,4,6,\ldots \\ (-1)^{\frac{n-1}{2}}\sin\left(s\,\frac{1+b}{2}\right), & n = 1,3,5,7,\ldots, \end{cases} \tag{23}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eqs. (20) and (21) into Eqs. (12)–(15), respectively, Eqs. (12) and (14) can be automatically satisfied, respectively. Then the remaining Eqs. (13) and (15) reduce to the form after integration with respect to x in $[b, x]$, respectively:

$$\sum_{n=0}^{\infty} a_n B_n \int_0^{\infty} s^{-1}[1+f(s)]\, G_n(s)\, J_{n+1}\left(s\,\frac{1-b}{2}\right)[\sin(sx)-\sin(sb)]\,ds = \frac{\pi}{2\mu}\,\tau_0(1+\lambda)\,(x-b)\,, \tag{24}$$

$$\sum_{n=0}^{\infty}\left(b_n-\frac{e_{15}}{\varepsilon_{11}}\,a_n\right)B_n \int_0^{\infty} s^{-1}G_n(s)\, J_{n+1}\left(s\,\frac{1-b}{2}\right)[\sin(sx)-\sin(sb)]\,ds = -\frac{\pi D_0}{2\varepsilon_{11}}\,(x-b)\,, \tag{25}$$

where $\lambda = \dfrac{e_{15}D_0}{\varepsilon_{11}\tau_0}\,, \qquad f(s) = \dfrac{\gamma - s}{s}\,.$ (26)

The semi-infinite integral in Eqs. (24) and (25) can be modified as [5]

$$\begin{aligned}
&\int_0^{\infty}\frac{1}{s}\,[1+f(s)]\,J_{n+1}\left(s\,\frac{1-b}{2}\right)\cos\left(s\,\frac{1+b}{2}\right)\sin(sx)\,ds\\
&= \frac{1}{2(n+1)}\left\{\frac{\left(\dfrac{1-b}{2}\right)^{n+1}\sin\left(\dfrac{(n+1)\pi}{2}\right)}{\left\{x+\dfrac{1+b}{2}+\sqrt{\left(x+\dfrac{1+b}{2}\right)^2-\left(\dfrac{1-b}{2}\right)^2}\right\}^{n+1}} - \sin\left[(n+1)\sin^{-1}\left(\frac{1+b-2x}{1-b}\right)\right]\right\}\\
&\quad + \int_0^{\infty} s^{-1}f(s)\,J_{n+1}\left(s\,\frac{1-b}{2}\right)\cos\left(s\,\frac{1+b}{2}\right)\sin(sx)\,ds\,,
\end{aligned} \tag{27}$$

$$\begin{aligned}
&\int_0^{\infty}\frac{1}{s}\,[1+f(s)]\,J_{n+1}\left(s\,\frac{1-b}{2}\right)\sin\left(s\,\frac{1+b}{2}\right)\sin(sx)\,ds\\
&= \frac{1}{2(n+1)}\left\{\cos\left[(n+1)\sin^{-1}\left(\frac{1+b-2x}{1-b}\right)\right] - \frac{\left(\dfrac{1-b}{2}\right)^{n+1}\cos\left(\dfrac{(n+1)\pi}{2}\right)}{\left\{x+\dfrac{1+b}{2}+\sqrt{\left(x+\dfrac{1+b}{2}\right)^2-\left(\dfrac{1-b}{2}\right)^2}\right\}^{n+1}}\right\}\\
&\quad + \int_0^{\infty} s^{-1}f(s)\,J_{n+1}\left(s\,\frac{1-b}{2}\right)\sin\left(s\,\frac{1+b}{2}\right)\sin(sx)\,ds\,.
\end{aligned} \tag{28}$$

For a large s, the integrands of the semi-infinite integral in Eqs. (27) and (28) are almost $1/s^2$, so the semi-infinite integral in Eqs. (27) and (28) can be evaluated numerically by Filon's method [1]. Thus, the semi-infinite integral in Eqs. (24) and (25) can be evaluated directly. Equations (24) and (25) can now be solved for the coefficients a_n and b_n by Schmidt's method

[10]. For brevity, Eq. (24) can be rewritten as (Eq. (25) can be solved using a similar method as following)

$$\sum_{n=0}^{\infty} a_n E_n(x) = U(x)\,, \quad b < x < 1\,, \tag{29}$$

where $E_n(x)$ and $U(x)$ are known functions, and the coefficients a_n are unknown and will be determined. A set of functions $P_n(x)$ which satisfy the orthogonality condition

$$\int_b^1 P_m(x)\,P_n(x)\,dx = N_n\delta_{mn}\,, \qquad N_n = \int_b^1 {P_n}^2(x)\,dx \tag{30}$$

can be constructed from the function $E_n(x)$, such that

$$P_n(x) = \sum_{i=0}^{n} \frac{M_{in}}{M_{nn}}\,E_i(x)\,, \tag{31}$$

where M_{ij} is the cofactor of the element d_{ij} of D_n, which is defined as

$$D_n = \begin{bmatrix} d_{00}, d_{01}, d_{02}, \ldots, d_{0n} \\ d_{10}, d_{11}, d_{12}, \ldots, d_{1n} \\ d_{20}, d_{21}, d_{22}, \ldots, d_{2n} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ d_{n0}, d_{n1}, d_{n2}, \ldots, d_{nn} \end{bmatrix}, \qquad d_{ij} = \int_b^1 E_i(x)\,E_j(x)\,dx\,. \tag{32}$$

Using Eqs. (29)–(32), we obtain

$$a_n = \sum_{j=n}^{\infty} q_j\,\frac{M_{nj}}{M_{jj}} \tag{33}$$

$$\text{with} \quad q_j = \frac{1}{N_j}\int_b^1 U(x)\,P_j(x)\,dx\,. \tag{34}$$

4 Intensity factors

The coefficients a_n and b_n are known, so that the entire stress field and the electric displacement can be obtained. However, in fracture dynamic mechanics, it is of importance to determine the stress τ_{yz} and the electric displacement D_y in the vicinity of the crack's tips. τ_{yz} and D_y along the crack line can be expressed respectively as

$$\tau_{yz}(x,0,t) = -\frac{2}{\pi}\sum_{n=0}^{\infty}(c_{44}a_n + e_{15}b_n)\,B_n\int_0^{\infty}[1+f(s)]\,G_n(s)\,J_{n+1}\left(s\,\frac{1-b}{2}\right)\cos(xs)\,ds\,, \tag{35}$$

$$D_y(x,0,t) = -\frac{2}{\pi}\sum_{n=0}^{\infty}(e_{15}a_n - \varepsilon_{11}b_n)\,B_n\int_0^{\infty}G_n(s)\,J_{n+1}\left(s\,\frac{1-b}{2}\right)\cos(xs)\,ds\,. \tag{36}$$

Observing the expression in Eqs. (35) and (36), the singular portion of the stress field and the singular portion of the electric displacement can be obtained respectively from the relationships [5]

$$\cos\left(s\,\frac{1+b}{2}\right)\cos(sx)=\frac{1}{2}\left\{\cos\left[s\left(\frac{1+b}{2}-x\right)\right]+\cos\left[s\left(\frac{1+b}{2}+x\right)\right]\right\},$$

$$\sin\left(s\,\frac{1+b}{2}\right)\cos(sx)=\frac{1}{2}\left\{\sin\left[s\left(\frac{1+b}{2}-x\right)\right]+\sin\left[s\left(\frac{1+b}{2}+x\right)\right]\right\},$$

$$\int_0^\infty J_n(sa)\cos(bs)\,ds=\begin{cases}\dfrac{\cos\left[n\sin^{-1}(b/a)\right]}{\sqrt{a^2-b^2}}\,, & a>b\\ -\dfrac{a^n\sin(n\pi/2)}{\sqrt{b^2-a^2}\left[b+\sqrt{b^2-a^2}\,\right]^n}\,, & b>a\end{cases},$$

$$\int_0^\infty J_n(sa)\sin(bs)\,ds=\begin{cases}\dfrac{\sin\left[n\sin^{-1}(b/a)\right]}{\sqrt{a^2-b^2}}\,, & a>b\\ \dfrac{a^n\cos(n\pi/2)}{\sqrt{b^2-a^2}\left[b+\sqrt{b^2-a^2}\,\right]^n}\,, & b>a\end{cases}.$$

The singular portion of the stress field and the singular portion of the electric displacement can be expressed respectively as follows:

$$\tau=-\frac{2}{\pi}\sum_{n=0}^\infty(c_{44}a_n+e_{15}b_n)\,B_nH_n(b,x)\,, \tag{37}$$

$$D=-\frac{2}{\pi}\sum_{n=0}^\infty(e_{15}a_n-\varepsilon_{11}b_n)\,B_nH_n(b,x)\,, \tag{38}$$

where $H_n(b,x)=-F_1(b,x,n)\,,\ n=0,1,2,3,4,5,\ldots$ (for $0<x<b$),

$$H_n(b,x)=(-1)^{n+1}F_2(b,x,n)\,,\quad n=0,1,2,3,4,5,\ldots,\quad (\text{for}\quad 1<x)\,,$$

$$F_1(b,x,n)=\frac{2(1-b)^{n+1}}{\sqrt{(1+b-2x)^2-(1-b)^2}\,[1+b-2x+\sqrt{(1+b-2x)^2-(1-b)^2}\,]^{n+1}}\,,$$

$$F_2(b,x,n)=\frac{2(1-b)^{n+1}}{\sqrt{(2x-1-b)^2-(1-b)^2}\,[2x-1-b+\sqrt{(2x-1-b)^2-(1-b)^2}\,]^{n+1}}\,.$$

At the left end of the right crack, we obtain the stress intensity factor K_L as

$$K_L=\lim_{x\to b^-}\sqrt{2\pi(b-x)}\cdot\tau=\sqrt{\frac{2}{\pi(1-b)}}\sum_{n=0}^\infty(c_{44}a_n+e_{15}b_n)\,B_n\,. \tag{39}$$

At the right end of the right crack, we obtain the stress intensity factor K_R as

$$K_R=\lim_{x\to b^+}\sqrt{2\pi(x-1)}\cdot\tau=\sqrt{\frac{2}{\pi(1-b)}}\sum_{n=0}^\infty(-1)^n\,(c_{44}a_n+e_{15}b_n)\,B_n\,. \tag{40}$$

At the left end of the right crack, we obtain the electric displacement intensity factor D_L as

$$D_L = \lim_{x \to b^-} \sqrt{2\pi(b-x)} \cdot D = \sqrt{\frac{2}{\pi(1-b)}} \sum_{n=0}^{\infty} (e_{15} a_n - \varepsilon_{11} b_n) B_n . \tag{41}$$

At the right end of the right crack, we obtain the electric displacement intensity factor D_R as

$$D_R = \lim_{x \to 1^+} \sqrt{2\pi(x-1)} \cdot D = \sqrt{\frac{2}{\pi(1-b)}} \sum_{n=0}^{\infty} (-1)^n (e_{15} a_n - \varepsilon_{11} b_n) B_n . \tag{42}$$

5 Numerical calculations and discussion

The dimensionless stress intensity factors K_L and K_R are calculated numerically. From the references (see e.g., [7], [8], [26], [27]), it can be seen that Schmidt's method is performed satisfactorily if the first ten terms of the infinite series to Eq. (29) are obtained. The solution of two collinear cracks of arbitrary length $a - b$ can easily be obtained by a simple change in the numerical values of the present paper $(a > b > 0)$, i.e., it can use the results of the collinear cracks of length $1 - b/a$ in the present paper. The solution of this paper is suitable for the arbitrary length of two collinear cracks. The results of the present paper are shown in Figs. 2 to 7. Form the results, the following observations are very significant:

(i) The dynamic stress intensity factors not only depend on the crack length, the electric loading and the frequency of the incident wave, but also depend on the shear stress wave velocity of the piezoelectric materials.

(ii) The effects of the two collinear cracks decrease when the distance between the two collinear cracks increases.

(iii) The stress intensity factor becomes big with increasing electric loading, in other words, the electric field will increase the magnitude of the stress intensity factor. This is due to the coupling between the electric and the mechanical fields.

(iv) The dynamic response of the electric field on the hand is independent of the external mechanical load. It is coherent with the applied dynamic elastic load.

(v) It can be concluded that the dynamic elastic field will promote the propagation of the crack at different stages of the loading process.

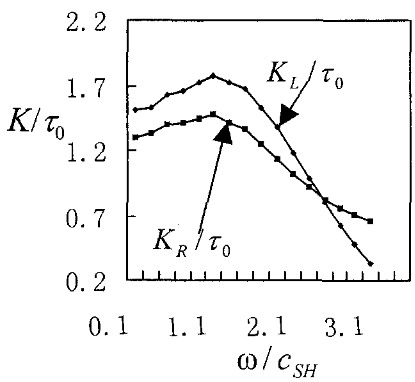

Fig. 2. Stress intensity factors versus ω/c_{SH}

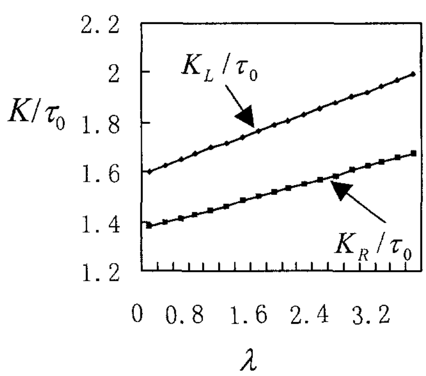

Fig. 3. Stress intensity factors versus λ for $b = 0.1, \omega/c_{SH} = 0.5$

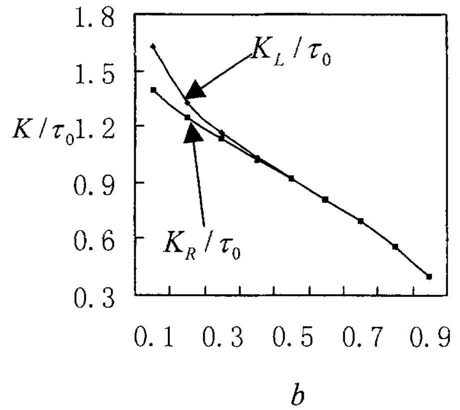

Fig. 4. Stress intensity factors versus b for $\lambda = 0.2, \omega/c_{SH} = 0.5$

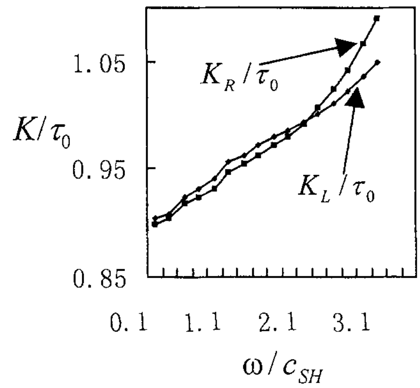

Fig. 5. Stress intensity factors versus ω/c_{SH}

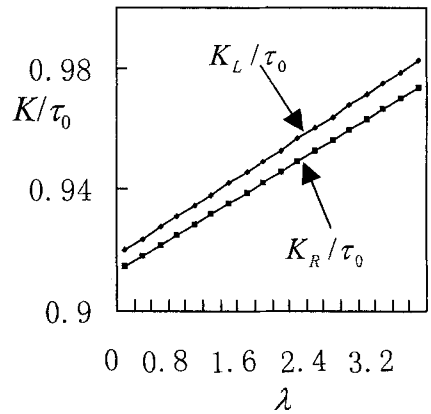

Fig. 6. Stress intensity factors versus λ for $b = 0.5, \omega/c_{SH} = 0.5$

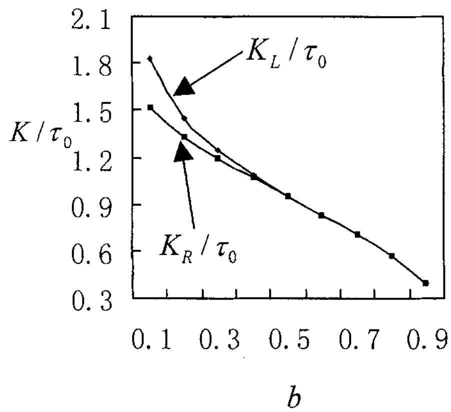

Fig. 7. Stress intensity factors versus b for $\lambda = 0.2, \omega/c_{SH} = 1.0$

6 Conclusions

Fracture is one of the properties that limits the use of piezoelectric materials as sensors and actuators in smart material and structure technology. We developed an electro-elastic fracture mechanics theory to determine the singular stress and electric fields near the crack tip for piezoelectric materials having two finite cracks under longitudinal shear waves. The anti-plane electro-elastic problem of a piezoelectric material with two collinear cracks has been analyzed theoretically. The traditional concept of linear elastic fracture mechanics is extended to include the piezoelectric effects, and the results are expressed in terms of the stress intensity factors. The developed method is applied to illustrate the fundamental behavior of the interacting cracks in piezoelectric materials under dynamic loading. Furthermore, the effect of the geometry of the interacting cracks, the shear stress wave velocity of the piezoelectric materials, and the frequency of the incident wave upon the dynamic stress intensity factor of the crack are examined and their influence discussed. This study reveals the importance of the electro-mechanical coupling terms upon the resulting dynamic stress intensity factors.

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